

**THE FUNDAMENTALS OF IMAGING  
PHYSICS AND RADIOBIOLOGY**



Copy of Roentgen's radiograph of his wife's "hand with rings," made soon after his discovery of x rays in November 1895 in Wurzburg, Germany. (See Glasser O. *Dr. W. C. Roentgen*, p. 39, 2nd ed., Springfield, Charles C Thomas, 1958.)

NINTH EDITION

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**THE FUNDAMENTALS  
OF  
IMAGING PHYSICS  
AND  
RADIOBIOLOGY**

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*By*

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*Dedicated to my wife*



## PREFACE TO THE NINTH EDITION

A number of important changes have been made in this edition, the first being the obvious new title reflecting the demise of radium in medicine and its replacement by artificial radioisotopes (radionuclides).

To improve readability, a double-column format has been adopted, serving to bring more illustrations closer to their explanation in the text. A helpful addition should be the outline of the contents of each chapter below its heading.

Greater attention is paid to high-frequency generation of x rays. Because this involves microprocessors (computers), which also find wide application in the department generally, I have added a chapter on basic computer science. A typical modern control panel for comparison with the primitive type indicates how much progress has occurred in x-ray production and control.

The rapid increase in the number of approved mammography centers demands a corresponding need for specialized technologists. I have therefore updated the requirements for mammographic equipment and included a section outlining quality assurance, especially as it applies to the responsibilities of the designated mammographer. A separate chapter is now devoted to mammography.

Digital imaging has been assigned a new chapter to include more information about subtraction angiography.

A new chapter takes up the basic science of radiobiology: the effects of radiation, especially of x rays, on cells, tissues, and organs. Since harmful effects are dose-

dependent, this fits well with the chapter on health physics, which has been updated.

More questions and problems, with sample solutions, have been added. Besides, the index continues to be user-friendly to facilitate the search for answers in the text.

Many thanks are due Joe Burrage (Diagnostic Imaging, Inc.) for help with modern x-ray generator circuitry in the form of block diagrams, as well as explanations of newer equipment and devices. I am also grateful to Ted Kosnik, Ph.D., for his answers about matters physics, and for plotting the attenuation curves of 100-kV x rays; and to my son Jay E. Selman, M.D., for his review of the chapter on computer science and his very helpful comments.

I am also indebted to the following individuals and companies, thanking them for providing brochures and technical information on equipment and supplies: Larry Spittler, RT (Diagnostic Imaging, Inc.); Nu-Tech (Tyler, Texas); and Paul Oster, RT, CNMT, Chief Technologist of the Nuclear Medicine Laboratory, East Texas Medical Center Hospital, Tyler, Texas.

New illustrations and modifications of previous ones have been meticulously executed by artist Gene Johnson (Tyler, Texas), and for this I am most grateful. At this point I wish to thank again the retired artist, Howard Marlin, for his excellent illustrations in the first eight editions of this book, dating back to 1954, and carried over to the ninth edition.

Finally, and by no means least, I would like to express my sincere gratitude to Michael Thomas, head of Charles C Thomas, Publisher, and his most competent

staff, and to Claire Slagle, the superb editor of my book for its Ninth Edition.  
assigned to me, for their cooperation and  
diligent attention to details in the publication

Joseph Selman, M.D.



## INTRODUCTION

As a student entering an approved School of Radiologic Technology, you have probably heard about the hazards of overexposure to x rays, a special form of light. The key word here is “approved,” a designation assuring that the school has in place not only the required teaching program, but also all necessary protective measures to keep your occupational exposure to x rays and related radiation within acceptable limits. Necessary protective maintenance has been established by regulations (“regs”) mandated by the relevant State Agency and the U.S.A. Nuclear Regulatory Commission (NRC). I shall briefly summarize here the basic protective features that you will find in a modern radiology department.

The walls of the x-ray rooms have built-in lead or equivalent protection in accordance with State regulations, and have been tested by a certified Health Physicist or other qualified person. A number of protective features have been incorporated in the x-ray equipment as mandated by the Bureau of Radiological Health (BRH). You will note that the console, which controls x-ray tube operation by the radiographer, is located inside a control booth whose walls contain a prescribed thickness of lead (metal) for the radiographer’s protection; even the booth’s window consists of protective glass of proper

thickness.

In fluoroscopy you will see a lead-containing rubber or plastic curtain hanging from the viewing assembly to protect the fluoroscopist; leaded aprons and gloves are also available for your personal protection.

The radiographer must follow all rules governing radiation protection according to the three basic principles: distance, shielding, and time: radiation dose decreases rapidly with increasing distance from the x-ray source; shielding of the hands and body by lead or other protective material; and reducing the time spent in the area during exposure all contribute greatly to dose reduction. Finally, special badges must be worn by all personnel in the x-ray area to detect any breach in safe operating procedures.

In my experience over many years, monitored exposures of radiographers have, with rare exceptions, been well within prescribed limits. Moreover, the risk from occupational exposure to radiation in the radiology department is trivial compared to other occupational, recreational, and highway travel risks. The same applies to the risk in a modern radiology department.

The chapter on Health Physics, which you will study later, provides additional details on the subject of radiation protection of personnel and patients.



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## Chapter 1

# SIMPLIFIED MATHEMATICS

### Topics Covered in This Chapter

- Arithmetic
  - Fractions
  - Percent
  - Decimal Fractions
  - Significant Figures
- Algebra
  - Ratio and Proportion
  - Plane Geometry
  - Graphs and Charts
  - Large and Small Numbers
  - Logarithms
  - Questions and Problems

**A**LL OF THE PHYSICAL SCIENCES have in common a firm basis in mathematics. This is no less true of radiologic physics, an important branch of the physical sciences. Clearly, then, in approaching a course in radiologic physics you, as a student technologist, should find your path smoothed by an adequate background in the appropriate areas of mathematics.

We shall assume here that you have had at least the required high school exposure to mathematics, although this may vary widely from place to place. However, realizing that much of this material may have become hazy with time, we shall review the simple but necessary aspects of arithmetic, algebra,

and plane geometry. Such a review should be beneficial in at least two ways. First, it should make it easier to understand the basic principles and concepts of radiologic physics. Second, it should aid in the solution of such everyday problems as conversion of radiographic techniques, interpretation of tube rating charts, determination of radiographic magnification, and many others that may arise from time to time.

The discussion will be subdivided as follows: (1) arithmetic, (2) algebra, (3) ratio and proportion, (4) geometry, (5) graphs and charts, and (6) large and small numbers. Only fundamental principles will be included.

### ARITHMETIC

Arithmetic is calculation or problem solving by means of definite numbers. We shall assume that you are familiar with addition, subtraction, multiplication, and division and shall therefore omit these operations.

#### Fractions

In arithmetic, *a fraction may be defined as one or more equal parts of a unit*. For example,  $\frac{1}{2}$ ,  $\frac{1}{3}$ , and  $\frac{2}{5}$  are fractions. The quan-

tity below the line is called the **denominator**; it indicates the number of equal parts into which the unit is divided. The quantity above the line is the **numerator**; it indicates the number of equal parts taken. Thus, if a pie were divided into three equal parts, the denominator would be 3; and if two of these parts were taken, the numerator would be 2, so, the two segments would represent  $\frac{2}{3}$  of the pie.

Fractions represent **the division of one quantity by another**. This extends the concept of fractions to expressions in which the numerator is larger than the denominator, as in the fraction  $\frac{5}{2}$ .

If the numerator is smaller than the denominator, as  $\frac{3}{5}$ , we have a **proper fraction**. If the numerator is larger than the denominator, as  $\frac{5}{3}$ , we have an **improper fraction**, because  $5 \div 3 = 1\frac{2}{3}$ , which is really an integer plus a fraction.

In **adding** fractions, all of which have the **same** denominator, add all the numerators first and then place the sum over the denominator:

$$\frac{2}{7} + \frac{3}{7} + \frac{6}{7} + \frac{5}{7} = \frac{2 + 3 + 6 + 5}{7} = \frac{16}{7}$$

$$\frac{16}{7} = 2\frac{2}{7}$$

**Subtraction** of fractions having identical denominators follows the same rule:

$$\frac{6}{7} - \frac{4}{7} = \frac{6 - 4}{7} = \frac{2}{7}$$

If fractions are added or subtracted, and the denominators are **different**, then the **least common denominator** must be found. This is the smallest number which is exactly divisible by all the denominators. Thus,

$$\frac{1}{2} + \frac{2}{3} - \frac{3}{4} = ?$$

The smallest number which is divided exactly by each denominator is 12. Place 12 in the denominator of a new fraction:

$$\frac{\quad}{12}$$

Divide the denominator of each of the fractions in the old equation into 12, and then multiply the answer by the numerator of that fraction; each result is then placed in the numerator of the new fraction:

$$\frac{6 + 8 - 9}{12} = \frac{5}{12}$$

Multiplying fractions means taking their product. To multiply fractions, take the product of the numerators and place it over the product of the denominators,

$$\frac{4}{5} \times \frac{3}{10} = \frac{4 \times 3}{5 \times 10} = \frac{12}{50}$$

The resulting fraction can be reduced by dividing the numerator and the denominator by the same number, in this case, 2:

$$\frac{12}{50} \div \frac{2}{2} = \frac{6}{25}$$

which cannot be further simplified.

Note that when the numerator and the denominator are both multiplied or divided by the same number, the value of the fraction does not change. For instance,

$$\frac{3}{5} \times \frac{2}{2} = \frac{6}{10}$$

$$\text{is the same as } \frac{3}{5} \times 1 = \frac{3}{5}$$

When two fractions are to be divided, as  $\frac{4}{5} \div \frac{3}{7}$ , the fraction that is to be divided is the **dividend**, and the fraction that does the dividing is called the **divisor**. In this case,  $\frac{4}{5}$  is the dividend and  $\frac{3}{7}$  the divisor. The rule is to invert the divisor (called "taking the reciprocal") and multiply the dividend by it:

$$\frac{4}{5} \div \frac{3}{7}$$

$$\frac{4}{5} \times \frac{7}{3} = \frac{28}{15} = 1\frac{13}{15}$$

## Percent

A special type of fraction, *percent*, is represented by the sign % to indicate that the number standing with it is to be divided by 100. Thus,  $95\% = \frac{95}{100}$ . We do not use percentages directly in mathematical computations, but first convert them to fractions or decimals. For instance,

$$\begin{aligned} 150 \times 40\% &\text{ is changed to} \\ 150 \times \frac{40}{100} &\text{ or } 150 \times \frac{2}{5} \\ \text{or } 150 \times 0.40. & \end{aligned}$$

All these expressions equal 60.

## Decimal Fractions

Our common method of representing numbers as multiples of ten is embodied in the *decimal system*. A *decimal fraction* has as its denominator 10, or 10 raised to some power such as 100, 1000, 10,000, etc. The denominator is symbolized by a dot in a certain position. For example, the decimal  $0.2 = \frac{2}{10}$ ;  $0.02 = \frac{2}{100}$ ;  $0.002 = \frac{2}{1000}$ , etc. Decimals can be multiplied or divided, but care must be taken to place the decimal point in the proper position:

$$\begin{array}{r} 2.24 \\ \times 1.25 \\ \hline 1120 \\ 448 \\ \hline 224 \\ \hline 2.8000 \end{array}$$

First, add the total number of digits to the right of the decimal points in the numbers being multiplied, which in this case turns out to be four. Then point off four places from the right in the answer to determine the correct position of the decimal point. The decimal system is used everywhere in science and in the vast majority of countries in daily life.

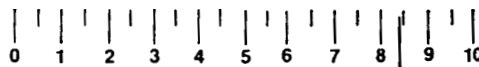


Figure 1.1. With this calibrated scale we can estimate to the nearest tenth. Thus, the position of the pointer indicates 8.4 units, the 0.4 being the last significant figure.

## Significant Figures

The precision (reproducibility of results) of any type of measurement is limited by the design of the measuring instrument. For example, a scale calibrated in grams as shown in Figure 1.1 allows an estimate to the nearest tenth of a gram. Thus, the scale in Figure 1.1 reads 8.4 grams. The last figure, 0.4, is estimated and is the *last significant figure*—that is, it is the last meaningful digit. Obviously, no greater precision is possible with this particular instrument. To improve precision, the scale would have to show a greater number of subdivisions.

Significant figures are used in various mathematical operations. For example, in addition:

item 1	98.26	grams
item 2	1.350	g
item 3	<u>260.1</u>	g
	359.710	g

Notice that three digits appear after the decimal point in the answer. But in item (3) there is only one digit, 1, after the decimal point; beyond this, the digits are unknown. Therefore, the digits after the 7 in the answer imply more than is known, since the answer can be no more precise than the least precise item being added. In this case, the answer should be properly stated as 359.7. In addition and subtraction the answer can have no more significant figures after the decimal point than the item with the *least number of significant figures after its decimal point*.

A different situation exists in multiplica-

tion and division. Here, the total number of significant figures in the answer equals that in the items having the **least total number of significant figures**. For example, in

$$\begin{array}{r} 25.23 \text{ cm} \\ \times 1.21 \text{ cm} \\ \hline 2523 \\ 5046 \\ \hline 2523 \\ \hline 30.5283 \text{ cm}^2 \end{array}$$

1.21 has fewer significant figures—three in all—so the answer should have three significant figures and be read as 30.5 (dropping

the 0.0283).

In general, to **round off** significant figures, observe the following rule: if the digit following the last significant figure is equal to or greater than 5, the last significant figure is increased by 1; if less than 5, it is unchanged. The rule is applied in the following examples:

$$\begin{array}{l} 45.157 \text{ is rounded to } 45.16 \\ 45.155 \text{ is rounded to } 45.16 \\ 45.153 \text{ is rounded to } 45.15 \end{array}$$

where the answer is to be expressed in four significant figures.

## ALGEBRA

The word **algebra**, derived from the Arabic language, connotes that branch of mathematics which deals with the relationship of quantities usually represented by letters of the alphabet—Roman, Greek, or Hebrew.

**Operations.** Mathematical operations with **letter symbols** are the same as with **numerals**, since both are symbolic representations of numbers which, in themselves, are abstract concepts. For example, the concept “four” may be represented by 4,  $2^2$ ,  $2 \times 2$ ,  $2 + 2$ , or  $3 + 1$ ; or by the letter  $x$  if the value of  $x$  is specifically designated to represent “four.” In algebra, just as in arithmetic, the fundamental operations include addition, subtraction, multiplication, and division; and there are fractions, proportions, and equations. Algebra provides a method of finding an unknown quantity when the relationship of certain known quantities is specified.

Algebraic operations are indicated by the same symbols as in arithmetic:

$$\begin{array}{l} + \text{ (plus) add} \\ - \text{ (minus) subtract} \\ \times \text{ (times) multiply} \\ \div \text{ (divided by) divide} \end{array}$$

= equals

To indicate **addition** in algebra, use the general expression

$$x + y \quad (1)$$

The symbols  $x$  and  $y$ , called **variables**, may represent any number or quantity. Thus, if  $x = 4$ , and  $y = 7$ , then, substituting these values in equation (1),

$$4 + 7 = 11$$

Similarly, to indicate **subtraction** in algebra, use the general expression

$$x - y$$

If  $x = 9$  and  $y = 5$  then

$$9 - 5 = 4$$

Notice that algebraic symbols may represent whole numbers, fractions, zero, and negative numbers, among others. Negative numbers are those whose value is less than zero and are designated as  $-x$ . In algebraic terms, add a positive and a negative number as follows:

$$x + -y$$